Graded Representations of Current Algebras

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Interactions of Quantum Affine Algebras with Cluster Algebras, Current Algebras and Categorification

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Current Algebra

Definition

Let $\mathfrak{g}$ be a simple Lie algebra over $\mathbb{C}$. The current algebra $\mathfrak{g}[t]$ is the vector space $\mathfrak{g} \otimes \mathbb{C}[t]$ with bracket

$$[a \otimes f, b \otimes g] = [a, b] \otimes fg \quad \forall a, b \in \mathfrak{g}, \ f, g \in \mathbb{C}[t].$$
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$\mathfrak{g}[t]$ inherits a grading from $\mathbb{C}[t]$.

Let $a_r := a \otimes t^r$. 
We can turn any representation of $\mathfrak{g}$ into a representation of $\mathfrak{g}[t]$ in the following manner:

If $V$ is a $\mathfrak{g}$-module and $z \in \mathbb{C}$, we define a $\mathfrak{g}[t]$-module structure on $V$ by:

$$a_r v = z r a_v.$$ 

for all $a \in \mathfrak{g}$, $v \in V$, and $r \in \mathbb{Z}^+$. This module is denoted $ev_z V$. $ev_z V$ is an irreducible $\mathfrak{g}[t]$-module if and only if $V$ is an irreducible $\mathfrak{g}$-module.
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$$a_r.v = z^r a.v \quad \forall a \in \mathfrak{g}, v \in V, r \in \mathbb{Z}_+.$$  

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$$a_r . v = z^r a . v \quad \forall a \in \mathfrak{g}, v \in V, r \in \mathbb{Z}_+.$$

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We are interested in graded representations of $g[t]$. 

**Definition**

A representation $V$ of $g[t]$ is called $\mathbb{Z}$-graded if it is a vector space which has the following properties:

$V = \bigoplus_{r \in \mathbb{Z}} V[r]$, $g \cdot V[r] \subseteq V[r+s]$, $g \in g$, $r \in \mathbb{Z}$, $s \in \mathbb{Z}^+$. 

Each $V[r]$ is a $g$-submodule of $V$. 

The only $\mathbb{Z}$-graded evaluation modules occur when $\mathbb{Z} = 0$. 
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Then, it is a simple exercise to show that $ev_0 V(n)$ are the graded irreducible representations of $\mathfrak{sl}_2[t]$. 
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For \( r, s \in \mathbb{Z}_+ \), let

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S(r, s) = \{ (b_p)_{p \geq 0} : b_p \in \mathbb{Z}_+, \sum_{p \geq 0} b_p = r, \sum_{p \geq 0} pb_p = s \}.
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Then, define

$$y(r, s) = \sum_{(b_p)_{p \geq 0} \in S(r, s)} y_0^{(b_0)} y_1^{(b_1)} \cdots y_s^{(b_s)}$$

where $y_j^{(p)} := \frac{y_j^p}{p!}$. 

$V(\xi)$ Modules
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For example, $y(2, 4) = y_0y_4 + y_1y_3 + y_2^{(2)}$. 


\( V(\xi) \) modules

Definition (Chari—Venkatesh, E. Feigin)

Let \( \xi = (\xi_1 \geq \cdots \geq \xi_m > 0) \) be a partition. Then, \( V(\xi) \) is the \( \mathfrak{sl}_2[t] \)-module generated by \( v_\xi \) with defining relations:
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x_r. v_\xi = 0
\]
\[
h_r. v_\xi = |\xi| \delta_{r,0} v_\xi, \quad \text{where } |\xi| = \sum_{j \geq 1} \xi_j
\]
\[
y_0^{|\xi|+1} v_\xi = 0
\]
\[
y(r, s) v_\xi = 0, \text{ where } s + r \geq 1 + rk + \sum_{j \geq k+1} \xi_j \text{ for some } k \in \mathbb{Z}_+.
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x_r \cdot v_\xi &= 0 \\
h_r \cdot v_\xi &= |\xi| \delta_{r,0} v_\xi, \quad \text{where } |\xi| &= \sum_{j \geq 1} \xi_j \\
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If \( \xi = 1^n \), then \( V(\xi) \cong W_{\text{loc}}(n) \).
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\end{align*}
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If $\xi = 1^n$, then $V(\xi) \cong W_{\text{loc}}(n)$.

If $\xi = \ell_2^m \ell_1$, then $V(\xi)$ is a Demazure module.
The motivation for finding a new presentation comes from the relations

\[ y(r, s)\nu_\xi = 0, \quad r + s \geq 1 + rk + \sum_{j \geq k+1} \xi_j \text{ for some } k \in \mathbb{Z}_+. \]
Motivation

The motivation for finding a new presentation comes from the relations

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1. The relationship between \( k \) and the pair \( r \) and \( s \) is unknown in general.
2. Notice that \( s \) only appears on the left hand side of the inequality.
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$$\nu_1 = 4$$
For \( i \geq 1 \), define \( \nu_i \) to be the number of parts of \( \xi \) greater than or equal to \( i \).

Let \( \xi = 4^2 21 \). Then,

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\begin{align*}
\nu_1 &= 4 \\
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\end{align*}
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Let $\xi = 4^221$. Then,

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\begin{align*}
\nu_1 &= 4 \\
\nu_2 &= 3 \\
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Let $\xi = 4^221$. Then,

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\nu_1 &= 4 \\
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\nu_3 &= 2 \\
\nu_4 &= 2 \\
\nu_j &= 0 \quad \forall j \geq 5
\end{align*}
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For $j \in \mathbb{N}$, we define a new partition $\xi^{(j)}$ given by

$$\xi^{(j)} := (\xi_1 \geq \xi_2 \geq \cdots \geq \xi_j \geq 0).$$
A Presentation of $\xi$ using $\xi^{tr}$

Theorem (M.)

The module $V(\xi)$ is isomorphic to the quotient of the local Weyl module $W_{\text{loc}}(|\xi|)$ by the $\mathfrak{sl}_2[t]$–submodule generated by the elements

$$\{ y(r, |(\xi^{tr})^{(r)}| - r + 1)w_{|\xi|} : 1 \leq r \leq \xi_1 - 1 \}.$$
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As a consequence of our proof, we have shown in $V(\xi)$ we have the relation

$$y(r, s)v_\xi = 0, \quad s \geq |(\xi^{tr})^{(r)}| - r + 1, \quad r \geq 1.$$
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y(r, s)v_\xi = 0, \quad s \geq |(\xi^{tr})^{(r)}| - r + 1, \quad r \geq 1.
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This shows that \( V(\xi) \) is finitely presented as a quotient of a local Weyl module.
Why find new presentations of $V(\xi)$?

We would like to understand the structure of $V(\xi)$ modules.
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Why find new presentations of $V(\xi)$?

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For example:
- When is $\text{Hom}(V(\xi), V(\xi')) \neq 0$?
- What is the kernel of $V(\xi) \rightarrow V(\xi')$ in this case?
Let $\xi = (\xi_1 \geq \xi_2 \geq \cdots \geq \xi_m > 0)$ be a partition. We begin by defining partitions $\xi^+, \xi^-$. 

If $m > 1$, then $\xi^- = (\xi^{-1} \geq \xi^{-2} \geq \cdots \geq \xi^{-m+2} \geq \xi^{-m+1} \geq 0)$ is given by $\xi^{-r} = \begin{cases} \xi_r & \text{if } r < m-1 \\ \xi_{m-1} - \xi_m & \text{if } r = m-1 \\ 0 & \text{if } r \geq m \end{cases}$ 

Define $\xi^+ = (\xi^1 \geq \xi^2 \geq \cdots \geq \xi^{m-1} \geq \xi^m \geq 0)$ is the unique partition associated to the $n$-tuple $(\xi_1, \xi_2, \ldots, \xi_{m-2}, \xi_{m-1} + 1, \xi_m)$. 
Let $\xi = (\xi_1 \geq \xi_2 \geq \cdots \geq \xi_m > 0)$ be a partition. We begin by defining partitions $\xi^+, \xi^-$. If $m > 1$, then $\xi^- = (\xi^-_1 \geq \xi^-_2 \geq \cdots \geq \xi^-_{m-2} \geq \xi^-_{m-1} \geq 0)$ is given by
Short Exact Sequences

Let $\xi = (\xi_1 \geq \xi_2 \geq \cdots \geq \xi_m > 0)$ be a partition. We begin by defining partitions $\xi^+, \xi^-$. If $m > 1$, then $\xi^- = (\xi^-_1 \geq \xi^-_2 \geq \cdots \geq \xi^-_{m-2} \geq \xi^-_{m-1} \geq 0)$ is given by

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Theorem (Chari—Venkatesh, 2014)

For $m > 1$, there exists a short exact sequence of $\mathfrak{sl}_2[t]$-modules

$$0 \rightarrow \tau_{(m-1)\xi_m} V(\xi^-) \xrightarrow{\varphi^-} V(\xi) \xrightarrow{\varphi^+} V(\xi^+) \rightarrow 0.$$
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With our new presentation, we provided a different construction of $\varphi^-$. 
Construction of $\varphi^-$

Let $\tilde{\xi}_1$ be the partition given by

$$\tilde{\xi}_1 = (\xi_1 \geq \xi_2 \geq \cdots \geq \xi_{m-2} \geq \xi_{m-1} - 1 \geq \xi_m - 1 \geq 0).$$
Construction of $\varphi^-$

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Lemma (M.)

There exists a well defined map of $\mathfrak{sl}_2[t]$–modules $V(\tilde{\xi}_1) \rightarrow V(\xi)$ extending the assignment $v_{\tilde{\xi}_1} \rightarrow y_{\nu_1(\xi)-1} v_{\xi}$. 
Construction of $\varphi^-$

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Then, $\varphi^-$ is a composition of the well defined maps from this lemma.
Short Exact Sequences

These short exact sequences can be used to provide additional information about $V(\xi)$. 

Chari and Venkatesh showed that $V(\xi)$ modules are fusion products. We also use them to study the tensor products of local Weyl modules and produce filtrations by Demazure modules.
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